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ABSTRACT

The purpose of this study was to find out what difficulties Nicaraguan children encounter in doing addition and subtraction exercises. The results were used in the design of radio lessons in mathematics for elementary school. The performance on addition exercises is analyzed with a view toward identifying the distinct skills that are required by the column-addition algorithm as it is used by children. Results suggest that carrying is a major contributor to difficulty and that passing (addition in which the leftmost column has a two-digit sum) is less difficult than carrying. (MP)

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Column Addition Skills

by

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In 1976 a paper-and-pencil test of column addition and subtraction was given to 2,873 children in the first through the sixth grades of public elementary schools in Managua, Nicaragua. The purpose of the study was to find out what difficulties Nicaraguan children encounter in doing addition and subtraction exercises; the results were used in the design of radio lessons in mathematics for elementary school. This study was a part of the research effort of the Radio Mathematics Project of Nicaragua, funded by the United States Agency for International Development and managed by the Institute for Mathematical Studies in the Social Sciences of Stanford University.

In this paper, the performance on addition exercises is analyzed with a view toward identifying the distinct skills that are required by the column-addition algorithm as it is used by children. The conclusions are not likely to be perfectly applicable to American children because of the many differences between the American and Nicaraguan school systems; nevertheless, the results may bring into question some of the assumptions underlying the design of American mathematics curriculums.

Design of the Test

The test covered only the paper-and-pencil algorithms for column addition and subtraction of whole numbers. The basic concepts of addition and subtraction were not directly tested, or were there any items designed to test applications of addition and subtraction, the

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horizontal form of addition and subtraction exercises, or mental computations. Also, the children were required to write only the answers, not the exercises themselves, so the skills of reading and writing numerals were not tested.

The test included 300 items, although no individual child was given more than 30 of these. Half of the items were addition exercises and half were subtraction. Each half was divided into three groups, roughly corresponding to grade levels 1, 2, and 3,¹ as follows:

<u>Grade level</u>	<u>Addition</u>	<u>Subtraction</u>
First grade	50 items	50 items
Second grade	50 items	50 items
Third grade	50 items	50 items

Ten subtests of approximately equal difficulty were constructed by choosing five items from each of the above six cells. First graders were given only 10 items, all at the first-grade level. Second graders were given 20 items, including exercises at both first-grade and second-grade levels. Students in the third through sixth grades did 30 items each, covering the first three grade levels.

In designing the test, our goal was to test as completely as we could every skill that might be needed in column addition or subtraction of whole numbers. To this end we first did a task analysis of the two algorithms, listing all the skills that would be needed, and then we partitioned the sets of addition and subtraction exercises in accordance with the skills and combinations of skills required in their solutions. In the following, only the design of the addition section will be discussed.

¹ We are referring here to Nicaraguan grade levels, not U.S. levels.

A task analysis is properly concerned with the process of doing the task and not just with characteristics of the task itself. Thus, if a problem admits of two different methods of solution, there are two appropriate task analyses. For example, some children may solve the following exercise working column by column, while others may simply count on from 345 to calculate the answer mentally:

$$\begin{array}{r} 345 \\ + 4 \\ \hline 349 \end{array}$$

In our analysis we did not incorporate alternate methods but assumed that most children would use the same methods.

Table 1 lists in abbreviated form the skills used as a basis for designing the test. The "short columns" referred to in the table occur in nonrectangular exercise like

$$\begin{array}{r} 74 \\ + 2 \\ \hline \end{array}$$

The blank space below the 7 can be interpreted as a zero, so that the exercise can be worked as: 4 plus 2 equals 6, 7 plus 0 equals 7.

In exercises like

$$\begin{array}{r} 7428 \\ + 2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ 53 \\ + 734 \\ \hline \end{array} \quad \begin{array}{r} 17 \\ 1 \\ + 613 \\ \hline \end{array}$$

there are several blank spaces distributed in different patterns.

We defined the variable to be the number of short columns. (As you will see below, this perception of the source of the difficulty was incorrect.)

The term "passing" pertains to exercises like

$$\begin{array}{r} 80 \\ + 70 \\ \hline 150 \end{array} \quad \begin{array}{r} 721 \\ + 525 \\ \hline 1246 \end{array}$$

in which the leftmost column has a two-digit sum. Carrying was defined in the usual way.

It should be noted that we did not consider regrouping in the ones column to be distinct from regrouping in the tens column. There are other places in which our list of skills might be considered incomplete. For example, we did not distinguish between contiguous and noncontiguous carries, and although we know that some digits are easier to read and write than others, we did not distinguish between them. More seriously, we treated all basic facts as equivalent even though it is probable that different skills are used in exercises such as $3 + 1$, $3 + 5$, and $9 + 8$; also, facts involving zeros, such as $8 + 0$ and $0 + 4$, were grouped with other basic facts. In compensation, in generating test items, we took these other possibilities into account by randomly choosing the digits, the position of carries, etc.

Using the variables given in Table 1, we partitioned the set of addition exercises of up to 3 columns and 4 addends into 124 classes from which we drew the items for testing. This classification takes into account the the skills required (as we had identified them) and the number of times each skill was needed, but it did not take into account the sequence in which the skills are used. Thus, we distinguish between

$$\begin{array}{r} 348 \\ +759 \\ \hline 1107 \end{array} \quad \text{and} \quad \begin{array}{r} 348 \\ +739 \\ \hline 1087 \end{array} ,$$

because one exercise has two carries and the other has only one, but we do not distinguish between

$$\begin{array}{r} 699 \\ 309 \\ + 28 \\ \hline 1036 \end{array} \quad \text{and} \quad \begin{array}{r} 699 \\ 390 \\ + 82 \\ \hline 1171 \end{array} .$$

(Both of these exercises require both carrying 1 and carrying 2, but the order is different.) In choosing the items for the test, we sampled each of the 124 classes of exercises but chose more often from the simpler types since the results are more readily analyzable when there are fewer complications present.

In choosing the items for the subtests we took care to distribute the different types of exercises as equally as possible. Thus, for example, there were 9 exercises with carrying in Subtests A, B, C, and E; 10 exercises with carrying in Subtests D, F, G, H, and I; and 11 in Subtest J.

Administration of the Test

The test was administered to 2,872 students in 75 classrooms in eight schools in Managua, Nicaragua (see Table 2). The schools were selected at random from among the public elementary schools of that city. Private schools, both sectarian and nonsectarian, were excluded, thereby excluding from the sample most children of wealthy families. The very poor would necessarily be excluded because they can rarely afford to send their children to school. There was nevertheless a wide range of socioeconomic classes represented--there were children of college professors as well as of unskilled laborers.

The testing of the first through fourth grades took place between August 16 and 30, 1976. We had originally planned to give the test only through the fourth grade, assuming that the simple skills required in the test would be so nearly mastered by the fourth grade that testing in higher grades would give us no further useful information. A glance at the children's work sheets showed that this was not true, especially for subtraction, so we returned to the same schools and tested both fifth and sixth grades. This additional testing ⁶was finished by mid-September;

hence, all testing took place during the seventh month of the school year. (School starts in Nicaragua about the middle of February.)

In the classrooms the test was administered by project personnel rather than by regular classroom teachers, so that we could better control the testing procedure. All 10 subtests were used in every classroom and were distributed in such a way that children sitting side by side would have different subtests, thereby reducing the likelihood of copying. No time limit was imposed, and the children were instructed to work carefully.

Analysis of Performance Data

In the following sections the data are presented in detail, and we discuss the exercises in terms of their relative difficulty to see what support, if any, the data give to the previously presented definitions of skills. A study of exercise difficulty alone is not sufficient to answer all of the questions we have about how students do exercises, but it can give us some indication of whether or not the postulated skills affect exercise difficulty. Two kinds of inferences can be made. First, if we have two types of exercises that differ in only one postulated skill and we find that the two types of exercises are equally difficult, we suspect either that the postulated skill is not in fact needed or that it has already been acquired by all students. The converse of the situation occurs when different performance levels are observed within a class of exercises that were predicted to be homogeneous. If this difference in performance can be related to some intrinsic characteristics of the exercises, then we can infer that a previously unrecognized skill is involved. This latter argument is the

stronger of the two, so we looked particularly for cases where there is an observed difficulty that the skills analysis does not explain.

A study of the data revealed that our skills analysis was deficient in both respects: We had defined some skills that clearly were not distinct from one another and we found several cases where we had to postulate additional skills. The new skills analysis developed from this study leads to a different classification of exercises than we had used in designing the test; the data presented in Appendix A are classified in accordance with the new analysis.

To simplify the discussion of the details of the analysis, the conclusions are presented before the supporting evidence is discussed.

Preview of Conclusions

A study of the addition data revealed the following deficiencies in the postulated skills analysis:

1. The number of columns makes no difference when there is more than one column. Although there is a significant difference in difficulty between one-column exercises and two-column exercises, there is no difference between those with two and those with three columns.
2. More addends do not increase exercise difficulty.
3. If there are zeros in an exercise, the exercise is significantly more difficult. Furthermore, combinations of the form $0 + N$ (where $1 \leq N \leq 9$) are more difficult than combinations of the form $N + 0$. However, the number of columns with zeros is not an important factor.
4. The characterization of short columns is inadequate.

On the positive side, we found evidence to support the following hypotheses:

1. Carrying is a major contributor to difficulty.
2. Carrying a number greater than 1 is more difficult than carrying 1.
3. Passing is less difficult than carrying.

We did not find sufficient evidence to either support or reject the hypotheses that increasing the number of carries increases the difficulty of the exercise, and that passing causes additional difficulty.

Data Analysis

The data were analyzed by means of t tests using the formula

$$t = \sqrt{\frac{p_1 - p_2}{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}}$$

where p_1 and p_2 are the observed proportions of correct responses to two sets of exercises, n_1 and n_2 are the numbers of children who were given the two sets, $q_1 = 1 - p_1$, and $q_2 = 1 - p_2$. In each case the comparison was made by varying only the exercise characteristic under consideration, holding all other characteristics constant.

Thus, in the analysis of the difference between two and three addends, five different cases were considered; in the first case all exercises had only one column and the other exercise characteristic variables had values of zero, in the second case passing was required in all exercises but no other skills were required, etc. (See Table A1 in Appendix A for a complete description of the characteristics of the exercises in the pairs of sets used in investigating the

difference between two and three addends.) By holding all other variables constant we were able to get a clear picture of the contribution to exercise difficulty of the characteristic in question. Also, independent comparisons were made for each grade so that learning gains would not confound the comparisons. Thus, in Table A1 for each of the five cases mentioned there are six comparisons, one for each grade.

The comparisons were made in the order in which they appear in Appendix A. At each stage whenever no significant difference was found the distinction was dropped in subsequent analyses. Thus, after the comparisons of two, three, and four addends, shown in Tables A1 and A2, showed that there was no difference in difficulty attributable to the number of addends the distinction between two, three, and four addend was discontinued and in further analyses these exercises were grouped together. In most of the first nine sets of comparisons (Tables A1 to A9) no significant differences were found.

Some rather distinctive patterns were noted throughout. The highest t values tended to occur at the lowest grade levels; this is most noticeable in Tables A9 and A15. This is undoubtedly due to the learning that occurs over grades; as a skill is mastered the difference in difficulty becomes less and less. A second common pattern is that the t values are highest in the first cases listed in the table; see especially Tables A14, A15, and A17. In general the cases are arranged in ascending order of difficulty and we believe that there is a masking effect in the later cases, where the requirement for other, possibly more difficult skills overshadows the effect of the skill under investigation. There are several exceptions to both of these general patterns, as can be seen in Tables A5 and A6.

Addends

In the comparison of two and three addends shown in Table A1, there are no values of t greater than 1.75. Since the value of t must be about 2.5 to be significant at the .01 level, this is not a strong indication of a significant difference. However, even more compelling evidence that three addends are not more difficult than two is that in six of the comparisons the value is quite large negatively (down to as much as -4.59). These large negative values indicate that there is a significant difference in the reverse direction, from which we could conclude the converse of the hypothesis being tested: There is indeed a difference between two and three addends, but two-addend exercises are more difficult.

In interpreting this finding it is necessary to keep in mind the conditions under which the comparisons were made. The most important of these conditions is that by controlling the values of all variables not under investigation we have effectively controlled the column sum; for example, when there is no passing and no carrying there can be no column whose sum is greater than 9. In the exercises with three addends this necessarily restricts the sizes of the addends, so that on the average the addends in a three-addend column are less than the addends in a two-addend column. With this in mind, we draw the conclusion that there is basically no difference between two and three addends for the population tested. Of course, we did not give the test in the first grade until the seventh month of the school year and it is possible that these children encountered some difficulty with three addends earlier in the year. If that were the case, it seems that whatever difficulty there is is mastered early.

Table A2 of Appendix A shows the comparisons made for three and four addends. Only in the first-grade comparison of the first case is there a significant difference (in either direction). This is a slight indication that more addends cause some difficulty in first grade, but there is no evidence that the difficulty is present beyond the first-grade level. Since the total evidence for the hypothesis that a greater number of addends causes a greater difficulty is so slight, we henceforth consider two, three, and four addends to be equivalent (and in further analyses will group them together).

Columns

We looked next at the difference between two and three columns. In Table A3 we show the results for six different cases, with a total of 27 different comparisons. In every comparison the absolute value of t was less than 2, and there was no consistency in the sign. The evidence is overwhelmingly in favor of the null hypothesis: The number of columns per se does not contribute to exercise difficulty. Since this finding is contrary to popular opinion, it behooves us to comment on it. We have no doubt that the average three-column exercise is more difficult than the average two-column exercise. For example, if the digits are chosen randomly, there is a higher probability of carrying in three-column exercises, there is a greater chance of encountering zeros, and it is only with three columns that one can find two occurrences of carrying. For these reasons, the commonly held notion that three-column exercises are more difficult is well justified. What we contend is that the difficulties arise from these other sources and are not caused by the number of columns per se.

The question then arises, is there any difference in difficulty between one-column and two-column exercises? The analysis of those differences is shown in Table A9. Four cases were considered for each of the six grades. In every case, at the first-grade level the differences were significant ($t = 9.02, 3.66, 3.52, \text{ and } 3.62$). For second grade, three of the four comparisons yielded a significant difference, but beyond that there were no significant differences. Thus, we conclude that there is a true difference in difficulty between one-column exercises and those with more than one column and that the skill of dealing with multiple columns is mastered by the third grade.

It seems then that our original characterization of the skill needed to deal with multiple columns was somewhat in error in that the difficulty appears to depend only upon the presence of more than one column and not upon the total number of columns. It might be best to think of the skill as a "reiteration" skill. A child must learn to work column by column rather than addend by addend, and once he has learned this concept he applies it immediately to any number of columns.

Short Columns

One of the more interesting outcomes of the study was a characterization of the difficulty connected with exercises with short columns. It has been known for some time that "nonrectangular" exercises are more difficult than "rectangular" ones. In first considering the difficulty we focused on the blank spaces in these exercises as the source of the difficulty, believing that these were somehow conceptually equivalent to zeros (in fact, we even called them "implicit zeros") and thought that the applicable exercise characteristic variable would be some function of the number and location of these blanks. In

designing the test we used an explicit function that depended only upon the number of columns with blank spaces rather than upon the total number of blanks or upon their location. However, in generating the items we took care to vary the total number of blanks and the locations of these so that we could detect more subtle effects. In looking closely at the data we found our first view of the source of difficulty to be quite mistaken.

We found that nonrectangular exercises are more difficult than rectangular exercises, as we had expected, but the variance in the proportions correct for the nonrectangular exercises was very large. Looking more closely we found exercises such as

$$\begin{array}{r}
 63 \\
 + 2 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 551 \\
 + 37 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 312 \\
 21 \\
 + 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 3 \\
 204 \\
 + 12 \\
 \hline
 \end{array}$$

to be markedly more difficult than exercises like

$$\begin{array}{r}
 31 \\
 87 \\
 + 1 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 4 \\
 12 \\
 81 \\
 + 12 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 512 \\
 902 \\
 2 \\
 + 4 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 533 \\
 12 \\
 510 \\
 + 813 \\
 \hline
 \end{array}
 .$$

(The analysis is shown in Case 1 of Table A20.)

In fact, these latter exercises are not more difficult than rectangular exercises such as

$$\begin{array}{r}
 61 \\
 35 \\
 + 22 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 35 \\
 22 \\
 41 \\
 + 31 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 822 \\
 323 \\
 914 \\
 + 510 \\
 \hline
 \end{array}
 .$$

(See Table A19.)

The difference between the two types of nonrectangular exercises exemplified above is that exercises of the first type have at least one column that contains a single digit. We hypothesize that the difficulty is due to the child's conception of addition as an

aggrandizing process in which one joins two or more sets; the child does not conceive the join of a single set, or the sum of a single number. Examination of the wrong answers for these exercises supported this view. One of the most common errors was to add a number from another column, for example,

$$\begin{array}{r} 63 \\ + 2 \\ \hline 85 \end{array} \quad \begin{array}{r} 551 \\ + 37 \\ \hline 881 \end{array} .$$

Further evidence about the child's thinking in this case is the following quote from an interview with a third grader (translated from the Spanish):

Interviewer: . . . Remember, you are going to explain to me step by step everything you do to find the answer. (Interviewer dictates the exercise 78.
- 6 which the child writes correctly.)

Child: Now I say 8 minus 6 . . . 8 minus 6 . . . uh, 2. (Writes the 2.) Since I don't have another--what's it called?--another subtrahend, then here I have . . . we pretend the 6 is here (lightly sketching a 6 in the space under the 7), but it's invisible. So I say 7 minus 6 gives 1. (Writes the 1.)

Interviewer: And what answer did you get?

Child: Twelve.

Although this child was doing subtraction, I imagine his thinking is much the same in addition.

Another frequent error in addition was adding the single digit into the column on the right:

$$\begin{array}{r} 63 \\ + 2 \\ \hline 11 \end{array} \quad \begin{array}{r} 213 \\ 21 \\ + 40 \\ \hline 94 \end{array} \quad \begin{array}{r} 3 \\ 204 \\ + 12 \\ \hline 39 \end{array} .$$

Also, some students simply ignore the odd digit:

$$\begin{array}{r} 63 \\ + 2 \\ \hline 5 \end{array} \quad \begin{array}{r} 551 \\ + 37 \\ \hline 88 \end{array} \quad 15 \quad \begin{array}{r} 213 \\ 21 \\ + 40 \\ \hline 74 \end{array} .$$

All of these incorrect procedures seem to reflect the notion that one must add something to a number--or, if there is nothing to add, do nothing.

If this hypothesis is correct, there should be no difficulty when a carried number is added to the odd digit, as in

$$\begin{array}{r} 35 \\ + 6 \\ \hline 41 \end{array} \quad \begin{array}{r} 78 \\ + 7 \\ \hline 155 \end{array} \quad \begin{array}{r} 9 \\ 91 \\ + 6 \\ \hline 106 \end{array} .$$

Indeed we found that supposition to be true: The above exercises are no more difficult than

$$\begin{array}{r} 66 \\ + 25 \\ \hline \end{array} \quad \begin{array}{r} 19 \\ + 59 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ 36 \\ + 14 \\ \hline \end{array} .$$

(See Cases 4 and 5 of Table A19.)

The term "no add column" in Appendix A refers, then, to the columns in which there is a single digit not augmented by a carry. Nonrectangular exercises with no "no add" columns were grouped with the rectangular exercises since there was no significant difference in difficulty. The comparisons shown in Table A13 are based on this classification.

Zeros

One of the biggest oversights in the original task analysis was the skill of adding zeros. From the data analysis we distinguished difficulties at two levels, one with zeros in the form $N + 0$ as found in exercises like

$$\begin{array}{r} 57 \\ + 20 \\ \hline \end{array}$$

and the other with zeros in the form $0 + N$, where N is a non-zero digit as in

$$\begin{array}{r} 50 \\ + 27 \\ \hline \end{array} .$$

That both of these forms cause difficulty is shown in Tables A11 and A12, and that the second is significantly more difficult than the first is shown in Table A16.

Carrying

The most highly significant differences in difficulty were found with carrying, as shown in Tables A14 and A15, and, as we expected, there was a significant difference between carrying 1 and carrying a number greater than 1, as shown in Table A18.

Passing

Passing probably contributes to exercise difficulty, as shown in Table A10. Although passing and carrying are similar in that both arise when the column sum is 10 or greater, these two skills are distinct, as seen in Table A17. Since carrying a number greater than 1 was found to be more difficult than carrying 1, it might be expected that passing a number greater than 1 would be more difficult than passing 1; but this appears not to be the case, as shown in Table A8.

Other Results

One would assume exercises that require two or more applications of a certain skill to be more difficult than exercises that require the application of that skill only once. In investigating this question we found this not to be the case in most instances. For example, when adding zeros it seemed not to matter whether there were one or two columns with zero. (see Table A4). Likewise the skill with "no add" columns seemed to apply equally well to exercises with one

add" column and with two "no add" columns (see Table A5). The results for carrying twice were somewhat ambiguous: From Table A6 it appears that carrying 1 twice is more difficult than carrying 1 once, but in Table A7 there is no clear evidence that the same applies to carrying a number greater than 1.

A Revised View of the Skills

This study has given rise to a new view of the skills involved in column addition. The necessary skills can be described in the following terms:

1. Handling exercises with more than one column appears to be a unitary skill which we can call the "reiteration" skill, that is, the concept of working an exercise column by column. The evidence suggests that, once this concept is formed, the child can apply it immediately to exercises with any number of columns. A more refined study of individual concept formation could clarify this issue.
2. The skill of adding single digits reiteratively, as required when there are more than two addends, may be a distinct skill, but if so, it is learned early in first grade. No increase in difficulty with increasing addends was observed beyond first grade.
3. The skill of passing appears to be distinct from other skills but there is still the unresolved question of whether the observed difficulty arises solely from the fact that larger column sums are involved.
4. The skill of adding zeros, both in the form $N + 0$ and in the

form $0 + N$, is a distinct skill. Here again this skill might be best considered a concept rather than a skill, since there was no observed difference in difficulty when there were several occurrences in the same exercises. There was a marked difference in difficulty between the forms $N + 0$ and $0 + N$ in the data analyzed. This difference may be due to the teaching methods used in Nicaragua; it would be desirable to analyze similar data collected in the United States before drawing a firm conclusion about the inherent difficulty of $0 + N$ relative to $N + 0$.

5. The skill involved in handling a "no add" column seems also to be better characterized as a concept. It appears that once a child understands that the sum of a single digit is simply that digit itself he can apply the concept as often as necessary.
6. As expected, carrying was found to be a distinct skill, one that is imperfectly learned and frequently applied incorrectly. Also as expected, there was found to be a distinct skill involved in carrying a number greater than 1.

Implications for Curriculum Design

Several of the findings discussed in this paper could have immediate application in the design of mathematics curriculum for the schools. Perhaps the most important finding is the one related to "no add" columns. The skill involved in this task seems to have been overlooked by many textbook writers and is unknown to many, perhaps most, elementary school teachers. In our experiences in Nicaragua we found the skill to be readily taught by several methods, and it is likely that this skill is

largely unrecognized because it is so readily learned. Many children will form the concept spontaneously simply from seeing a few correctly worked examples, even without explicit instruction. Errors made by other children who have not spontaneously formed the concept could easily be discounted by a classroom teacher as "careless error" if she were not aware that a distinct skill is involved.

That there is a distinct skill involved in carrying a number greater than 1 might also be overlooked in much the same way. Since this skill is required only when there are three or more addends errors might be assumed to result from the large number of addends. In teaching, a special effort should be made to assure that children are carrying correctly when the column sum is 20 or more.

The findings reported here about the column-by-column "reiteration" skill might be interpreted to mean that the teaching of two-column and three-column exercises could be done simultaneously, or even that three-column and four-column exercises need never be presented. This would be a questionable interpretation of the results, since there are difficulties connected with three-column exercises that were not investigated as part of this study. The particular difficulties we have in mind are in connection with reading and writing numerals. The tasks we tested did not involve these skills, but in teaching one would ordinarily expect children to read the numbers they are adding and certainly to read their answers. Since reading three-digit numbers requires knowledge not needed for reading two-digit numbers, the teaching of three-column exercises should probably be deferred until the children are comfortable with the numbers themselves.

Table 1
Skills and Exercise Characteristic Variables
used in Test Design

Skills	Variable	Description
Using a reiterative (column) algorithm	COL	Number of columns
Adding three or four 1-digit numbers	ADN	Number of addends
Adding short columns	SHC	Number of columns in which the number of digits is less than the number of addends in the exercise
Passing	PAS	The tens digit of the sum of the leftmost column, including the number carried from the preceding column
Carrying 1	CR1	Number of columns not the leftmost in which the sum, including the number carried from the preceding column, is between 10 and 19.
Carrying a number greater than 1	CRM	Number of columns not the leftmost in which the sum, including the number carried from the preceding column, is 20 or more.

Table 2
 Number of Schools, Classrooms, and Children
 Participating in the Testing Program

Grade	Number of schools	Number of classrooms	Number of students enrolled	Number of students who took test
1	7	13	622	518
2	7	13	585	511
3	7	13	554	520
4	7	12	537	503
5	8	13	502	399
6	8	11	439	422
TOTAL		75	3,239	2,873

Appendix A: t Tests

Table A1

Comparison of Performance on Addition Exercises
with Two and Three Addends

	Two addends		Three addends		t value
	N ^a	p ^b	N	p	
Case 1: One column, no passing, no zeros					
Grade 1:	413	.74	53	.66	1.15
Grade 2:	407	.90	51	.92	-.52
Grade 3:	413	.94	49	.98	-1.72
Grade 4:	402	.96	51	.98	-1.03
Grade 5:	319	.95	37	1.00	-4.06
Grade 6:	331	.96	41	1.00	-3.57
Case 2: One column, passing 1, no zeros					
Grade 1:	257	.57	103	.59	-.31
Grade 2:	258	.83	98	.86	-.56
Grade 3:	261	.84	103	.97	-4.59
Grade 4:	251	.91	100	.97	-2.35
Grade 5:	202	.96	80	.97	-.23
Grade 6:	214	.94	76	1.00	-3.58
Case 3: One column, no passing, one occurrence of N + 0					
Grade 1:	53	.53	53	.49	.41
Grade 2:	52	.85	52	.71	1.75
Grade 3:	54	.85	54	.93	-1.34
Grade 4:	52	1.00	52	1.00	.00
Grade 5:	41	.95	41	1.00	-1.47
Grade 6:	46	.91	46	1.00	-2.13
Case 4: Two columns, no passing, no zeros, no carrying					
Grade 1:	53	.45	103	.41	.48
Grade 2:	52	.87	101	.79	1.30
Grade 3:	53	.85	101	.92	-1.25
Grade 4:	50	.92	100	.94	-.44
Grade 5:	42	.93	78	.96	-.66
Grade 6:	44	.95	80	.97	-.41

(Table 1A, cont.)

	Two addends		Three addends		t value
	N ^a	p ^b	N	p	
Case 5: Two columns, no passing, no zeros, one occurrence of carrying 1					
Grade 1:	103	.03	0	.00	no test
Grade 2:	199	.54	154	.50	.68
Grade 3:	208	.71	155	.80	-1.94
Grade 4:	201	.86	150	.94	-2.71
Grade 5:	163	.91	115	.90	.25
Grade 6:	159	.89	130	.95	-1.93

Note. There were no occurrences of 0 + N, no "no add" columns, and no occurrences of carrying a number greater than 1.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A2

Comparison of Performance on Addition Exercises
with Three and Four Addends

	Three addends		Four addends		t value
	N ^a	p ^b	N	p	
Case 1: One column, passing 1, no zeros					
Grade 1:	103	.59	52	.33	3.13
Grade 2:	98	.86	52	.83	.41
Grade 3:	103	.97	53	.94	.82
Grade 4:	100	.97	49	.98	-.38
Grade 5:	80	.97	39	.97	-.13
Grade 6:	76	1.00	45	.96	1.37
Case 2: One column, no passing, one occurrence of N + 0					
Grade 1:	53	.49	51	.41	.82
Grade 2:	52	.71	51	.80	-1.07
Grade 3:	54	.93	50	.98	-1.25
Grade 4:	52	1.00	50	1.00	.00
Grade 5:	41	1.00	40	1.00	.00
Grade 6:	46	1.00	42	.93	1.78
Case 3: Two columns, no passing, no zeros					
Grade 1:	103	.41	53	.34	.86
Grade 2:	101	.79	51	.76	.41
Grade 3:	101	.92	49	.90	.40
Grade 4:	100	.94	51	.94	.00
Grade 5:	78	.96	37	1.00	-1.81
Grade 6:	80	.97	41	1.00	-1.68
Case 4: Two columns, passing 1, no zeros					
Grade 1:	106	.42	101	.36	.96
Grade 2:	103	.77	103	.76	.17
Grade 3:	107	.84	106	.93	-1.94
Grade 4:	102	.91	102	.92	-.25
Grade 5:	80	.95	81	.92	.86
Grade 6:	90	.99	85	.95	1.58

Note. There were no occurrences of 0 + N, no "no add" columns, and no carrying.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A3

Comparison of Performance on Addition Exercises
with Two and Three Columns

	Two columns		Three columns		t value
	N ^a	p ^b	N	p	
Case 1: No zeros, no "no add" columns, no carrying					
Grade 1:	209	.40	48	.38	.29
Grade 2:	204	.80	51	.86	-1.02
Grade 3:	203	.90	52	.94	-1.10
Grade 4:	201	.94	50	.94	-.13
Grade 5:	157	.96	40	.93	.72
Grade 6:	165	.97	39	.97	.00
Case 2: One occurrence of N + 0, no "no add" columns, no carrying					
Grade 1:	52	.19	52	.35	-1.87
Grade 2:	103	.68	105	.69	-.17
Grade 3:	103	.91	106	.85	1.41
Grade 4:	99	.90	101	.94	-1.04
Grade 5:	79	.95	83	.94	.29
Grade 6:	87	.94	89	.96	-.47
Case 3: One occurrence of 0 + N, no "no add" columns, no carrying					
Grade 1:	51	.27	106	.26	.20
Grade 2:	48	.50	103	.60	-1.10
Grade 3:	52	.77	107	.64	1.81
Grade 4:	50	.84	102	.82	.39
Grade 5:	42	.93	80	.88	1.00
Grade 6:	38	.87	90	.88	-.09
Case 4: No zeros, one "no add" column, no carrying					
Grade 1:	52	.21	52	.17	.52
Grade 2:	53	.51	52	.54	-.31
Grade 3:	53	.66	53	.68	-.22
Grade 4:	51	.90	49	.92	-.35
Grade 5:	41	.83	39	.85	-.24
Grade 6:	45	.89	45	.98	-1.76
Case 5: No zeros, no "no add" columns, one occurrence of carrying 1					
Grade 1:	103	.03	0	.00	no test
Grade 2:	353	.52	200	.53	-.13
Grade 3:	363	.75	257	.75	.00
Grade 4:	351	.89	252	.91	-.84
Grade 5:	278	.91	198	.93	-.66
Grade 6:	289	.92	201	.93	-.43

(Table 3A, cont.)

	Two column		Three columns		t value
	N ^a	p ^b	N	p	
Case 6: No zeros, no "no add" columns, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	155	.37	0	.00	no test
Grade 3:	211	.64	152	.62	.32
Grade 4:	199	.87	151	.83	.98
Grade 5:	160	.86	116	.83	.63
Grade 6:	172	.96	127	.87	2.68

Note. There were no exercises with passing. There were 2 to 4 addends in all cases.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A4

Comparison of Performance on Addition Exercises
with One and Two Occurrences of $N + 0$

	One occurrence of $N +$		Two occurrences of $N + 0$		t value
	N^a	p^b	N	p	
Grade 1:	104	.29	53	.30	-.15
Grade 2:	252	.55	52	.69	-1.91
Grade 3:	312	.78	53	.94	-3.96
Grade 4:	302	.84	50	.92	-1.93
Grade 5:	236	.90	42	.88	.44
Grade 6:	252	.95	44	.93	.54

Note. All exercises required passing, but there were no zeros, no occurrences of $0 + N$, no "no add" columns, and no carrying. There were 2 or 3 columns and 2 to 4 addends.

^a N = number of children given the exercises.

^b p = observed proportion of correct responses.

Table A5

Comparison of Performance on Addition Exercises
with One and Two Occurrences of "No Add" Columns

	One "no add" column		Two "no add" columns		t value
	N ^a	p ^b	N	p	
Grade 1:	52	.17	52	.23	-.77
Grade 2:	52	.54	50	.44	1.02
Grade 3:	53	.68	51	.61	.74
Grade 4:	49	.92	50	.88	.67
Grade 5:	39	.85	38	.92	-.97
Grade 6:	45	.98	38	.95	.73

Note. There was no passing, no zeros, and no carrying. There were 3 or 4 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A6

Comparison of Performance on Addition Exercises
with One and Two Occurrences of Carrying 1

	One occurrence of carrying 1		Two occurrences of carrying 1		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	200	.53	308	.46	1.36
Grade 3:	257	.75	366	.62	3.61
Grade 4:	252	.91	355	.84	2.84
Grade 5:	198	.93	278	.86	2.43
Grade 6:	201	.93	298	.93	-.14
Case 2: Passing 1, no zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	103	.43	206	.38	.92
Grade 3:	159	.67	261	.64	.58
Grade 4:	152	.78	250	.80	-.49
Grade 5:	123	.90	196	.85	1.39
Grade 6:	129	.89	218	.92	-.67
Case 3: No passing, one occurrence of N + 0					
Grade 1:	0	.00	0	.00	no test
Grade 2:	48	.60	105	.51	1.10
Grade 3:	52	.73	160	.65	1.11
Grade 4:	50	.88	152	.82	1.02
Grade 5:	42	.90	121	.87	.50
Grade 6:	38	.95	136	.93	.56

Note. There were no occurrences of 0 + N, no "no add" columns, and no occurrences of carrying a number greater than 1. There were 3 or 4 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A7

Comparison of Performance on Addition Exercises
with One and Two Occurrences of Carrying a Number Greater Than 1

	One occurrence of carrying a number greater than 1		Two occurrences of carrying a number greater than 1		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	0	.00	0	.00	no test
Grade 3:	152	.62	155	.49	2.37
Grade 4:	151	.83	151	.74	1.78
Grade 5:	116	.83	123	.76	1.38
Grade 6:	127	.87	125	.80	1.37
Case 2: Passing 1, no zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	0	.00	0	.00	no test
Grade 3:	105	.59	155	.54	.76
Grade 4:	100	.78	150	.71	1.20
Grade 5:	82	.79	122	.75	.81
Grade 6:	83	.82	125	.86	-.62
Case 3: No passing, with zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	0	.00	0	.00	no test
Grade 3:	53	.57	53	.43	1.46
Grade 4:	50	.80	50	.70	1.16
Grade 5:	39	.82	39	.69	1.35
Grade 6:	44	.89	44	.84	.69

Note. There were no occurrences of 0 + N, no "no add" columns, and no occurrences of carrying 1. There were 3 or 4 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A8

Comparison of Performance on Addition Exercises
with Passing 1 and Passing a Number Greater than 1

	Passing 1		Passing a number greater than 1		t value
	N ^a	p ^b	N	p	
Case 1: One occurrence of N + 0, no carrying					
Grade 1:	51	.35	0	.00	no test
Grade 2:	201	.57	52	.56	.12
Grade 3:	263	.76	53	.77	-.18
Grade 4:	251	.82	49	.92	-2.15
Grade 5:	199	.90	39	.95	-1.20
Grade 6:	211	.95	45	.91	.81
Case 2: No zeros, one occurrence of carrying 1, no occurrences of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	154	.46	53	.55	-1.10
Grade 3:	211	.71	53	.72	-.20
Grade 4:	202	.79	51	.80	-.22
Grade 5:	163	.88	41	.78	1.37
Grade 6:	172	.90	45	.89	.12
Case 3: No zeros, no occurrences of carrying 1, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	101	.39	0	.00	no test
Grade 3:	314	.61	52	.56	.61
Grade 4:	301	.78	50	.74	.61
Grade 5:	245	.81	42	.69	1.61
Grade 6:	248	.81	38	.87	-1.03

Note. There were no occurrences of 0 + N and no "no add" columns.
There were 2 or 3 columns and 3 or 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A9

Comparison of Performance on Addition Exercises
with One Column and More than One Column

	One column		More than one column		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros					
Grade 1:	466	.73	257	.40	9.02
Grade 2:	458	.90	255	.81	3.09
Grade 3:	462	.94	255	.91	1.83
Grade 4:	453	.96	251	.94	1.32
Grade 5:	356	.96	197	.96	.05
Grade 6:	372	.97	204	.97	-.20
Case 2: Passing 1 or 2, no zeros					
Grade 1:	412	.54	207	.39	3.66
Grade 2:	408	.84	206	.76	2.23
Grade 3:	417	.89	213	.88	.10
Grade 4:	400	.94	204	.92	.87
Grade 5:	321	.96	161	.93	1.38
Grade 6:	335	.96	175	.97	-.66
Case 3: No passing, one occurrence of N + 0, no occurrences of 0 + N					
Grade 1:	157	.48	104	.27	3.52
Grade 2:	155	.79	208	.69	2.20
Grade 3:	158	.92	209	.88	1.21
Grade 4:	154	1.00	200	.92	4.16
Grade 5:	122	.98	162	.94	1.81
Grade 6:	134	.95	176	.95	-.02
Case 4: No passing, no occurrences of N + 0, one occurrence of 0 + N					
Grade 1:	52	.54	157	.26	3.62
Grade 2:	52	.63	151	.57	.83
Grade 3:	53	.74	159	.68	.86
Grade 4:	49	.90	152	.82	1.45
Grade 5:	39	.90	122	.90	.09
Grade 6:	45	1.00	128	.87	4.29

Note. There were no "no add" columns and no carrying. There were 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A10

Comparison of Performance on Addition Exercises
with Passing and No Passing

	With passing		No passing		t value
	N ^a	p ^b	N	p	
Case 1: One column, no zeros, no carrying					
Grade 1:	466	.73	412	.54	5.89
Grade 2:	458	.90	408	.84	2.76
Grade 3:	462	.94	417	.89	3.14
Grade 4:	453	.96	400	.94	1.62
Grade 5:	356	.96	321	.96	-.45
Grade 6:	372	.97	335	.96	.60
Case 2: Two or three columns, no zeros, no carrying					
Grade 1:	257	.40	207	.39	.21
Grade 2:	255	.81	206	.76	1.41
Grade 3:	255	.91	213	.88	.82
Grade 4:	251	.94	204	.92	.86
Grade 5:	197	.96	161	.93	.94
Grade 6:	204	.97	175	.97	.02
Case 3: Two or three columns, one occurrence of N + 0, no occurrences of 0 + N, no carrying					
Grade 1:	104	.27	104	.29	-.30
Grade 2:	208	.69	304	.56	3.04
Grade 3:	209	.88	365	.78	3.26
Grade 4:	200	.92	351	.85	2.70
Grade 5:	162	.94	275	.91	1.37
Grade 6:	176	.95	297	.95	.08
Case 4: Two or three columns, no occurrences of N + 0, one occurrence of 0 + N, no carrying					
Grade 1:	157	.26	52	.29	-.42
Grade 2:	151	.57	52	.63	-.83
Grade 3:	159	.68	53	.72	-.57
Grade 4:	152	.82	49	.80	.36
Grade 5:	122	.90	39	.74	2.05
Grade 6:	128	.87	45	.84	.55
Case 5: No zeros, one occurrence of carrying 1, no occurrences of carrying a number greater than 1					
Grade 1:	103	.03	0	.00	no test
Grade 2:	553	.52	412	.47	1.72
Grade 3:	620	.75	472	.71	1.55
Grade 4:	603	.90	455	.82	3.69
Grade 5:	476	.92	365	.90	.70
Grade 6:	490	.92	383	.92	.18

(Table A10, cont.)

	With passing		No passing		t value
	N ^a	p ^b	N	p	
Case 6: No zeros, no occurrences of carrying 1, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	155	.37	101	.39	-.21
Grade 3:	363	.63	366	.60	.94
Grade 4:	350	.85	351	.78	2.52
Grade 5:	276	.85	287	.79	1.58
Grade 6:	299	.92	286	.82	3.66

Note. There were no "no add" columns. There were 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table All
Comparison of Performance on Addition Exercises
with No Occurrences and One Occurrence of $N + 0$

	No occurrences of N + 0		One occurrence of N + 0		t value
	N ^a	p ^b	N	p	
Case 1: One column, no passing					
Grade 1:	466	.73	157	.48	5.63
Grade 2:	458	.90	155	.79	3.20
Grade 3:	462	.94	158	.92	1.07
Grade 4:	453	.96	154	1.00	-4.35
Grade 5:	356	.96	122	.98	-1.71
Grade 6:	372	.97	134	.95	.93
Case 2: Two or three columns, no passing, no "no add" columns, no carrying					
Grade 1:	257	.40	104	.27	2.41
Grade 2:	255	.81	208	.69	3.19
Grade 3:	255	.91	209	.88	.86
Grade 4:	251	.94	200	.92	.64
Grade 5:	197	.96	162	.94	.45
Grade 6:	204	.97	176	.95	1.09
Case 3: Two or three columns, passing 1, no "no add" columns, no carrying					
Grade 1:	207	.39	104	.29	1.78
Grade 2:	206	.76	252	.55	4.78
Grade 3:	213	.88	312	.78	3.15
Grade 4:	204	.92	302	.84	2.75
Grade 5:	161	.93	236	.90	1.01
Grade 6:	175	.97	252	.95	.94
Case 4: Two or three columns, no passing, one to two "no add" columns, no carrying					
Grade 1:	156	.20	53	.21	-.10
Grade 2:	155	.50	52	.60	-1.30
Grade 3:	157	.65	53	.77	-1.73
Grade 4:	150	.90	50	.92	-.44
Grade 5:	118	.87	42	.98	-3.00
Grade 6:	128	.94	44	.91	.61
Case 5: Two or three columns, no passing, no "no add" columns, one occurrence of carrying 1					
Grade 1:	103	.03	0	.00	no test
Grade 2:	553	.52	99	.61	-1.55
Grade 3:	620	.75	104	.76	-.22
Grade 4:	603	.90	100	.91	-.32
Grade 5:	476	.92	82	.87	1.24
Grade 6:	490	.92	77	.94	-.37

(Table All, cont.)

	No occurrences of N + 0		One occurrence of N + 0		t value
	N ^a	p ^b	N	p	
Case 6: Two or three columns, no passing, no "no add" columns, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	155	.37	0	.00	no test
Grade 3:	363	.63	102	.63	.09
Grade 4:	350	.85	101	.80	1.10
Grade 5:	276	.85	76	.87	-.54
Grade 6:	299	.92	85	.86	1.40

Note. There were no occurrences of 0 + N. There were 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A12

Comparison of Performance on Addition Exercises
with No Occurrences and One Occurrence of $0 + N$

	No occurrences of 0 + N		One occurrence of 0 + N		t value
	N ^a	p ^b	N	p	
Case 1: One column, no passing					
Grade 1:	466	.73	52	.54	2.64
Grade 2:	458	.90	52	.63	3.96
Grade 3:	462	.94	53	.74	3.34
Grade 4:	453	.96	49	.90	1.36
Grade 5:	356	.96	39	.90	1.14
Grade 6:	372	.97	45	1.00	-3.56
Case 2: Two or three columns, no passing, no "no add" columns, no carrying					
Grade 1:	257	.40	157	.26	2.98
Grade 2:	255	.81	151	.57	5.29
Grade 3:	255	.91	159	.68	5.49
Grade 4:	251	.94	152	.82	3.26
Grade 5:	197	.96	122	.90	1.91
Grade 6:	204	.97	128	.87	3.03
Case 3: Two or three columns, passing 1 or 2, no "no add" columns, no or carrying					
Grade 1:	207	.39	52	.29	1.38
Grade 2:	206	.76	52	.63	1.77
Grade 3:	213	.88	53	.72	2.48
Grade 4:	204	.92	49	.80	1.90
Grade 5:	161	.93	39	.74	2.63
Grade 6:	175	.97	45	.84	2.31
Case 4: Two or three columns, no passing, 1 to 2 "no add" columns, no carrying					
Grade 1:	156	.20	154	.19	.21
Grade 2:	155	.50	153	.49	.13
Grade 3:	157	.65	154	.65	.00
Grade 4:	150	.90	151	.79	2.68
Grade 5:	118	.87	116	.87	-.19
Grade 6:	128	.94	124	.87	2.01

(Table A12, cont.)

	No occurrences of 0 + N		One occurrence of 0 + N		t value
	N ^a	p ^b	N	p	
Case 5: Two or three columns, no passing, no "no add" columns, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	155	.37	0	.00	no test
Grade 3:	363	.63	53	.60	.46
Grade 4:	350	.85	49	.73	1.79
Grade 5:	276	.85	39	.69	2.01
Grade 6:	299	.92	45	.73	2.76

Note. There were no occurrences of N + 0 and occurrences of carrying

1. There were 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A13

Comparison of Performance on Addition Exercises
with No "No Add" Columns and 1 or 2 "No Add" Columns

	No "no add" columns		One to two "no add" columns		t value
	N ^a	p ^b	N	p	
Case 1: No zeros, no carrying					
Grade 1:	257	.40	156	.20	4.39
Grade 2:	255	.81	155	.50	6.74
Grade 3:	255	.91	157	.65	6.05
Grade 4:	251	.94	150	.90	1.25
Grade 5:	197	.96	118	.87	2.57
Grade 6:	204	.97	128	.94	1.26
Case 2: One occurrence of N + 0, no occurrences of 0 + N, no carrying					
Grade 1:	104	.27	53	.21	.85
Grade 2:	208	.69	52	.60	1.13
Grade 3:	209	.88	53	.77	1.79
Grade 4:	200	.92	50	.92	.01
Grade 5:	162	.94	42	.98	-1.27
Grade 6:	176	.95	44	.91	.81
Case 3: No occurrences of N + 0, one occurrence of 0 + N, no carrying					
Grade 1:	157	.26	154	.19	1.40
Grade 2:	151	.57	153	.49	1.31
Grade 3:	159	.68	154	.65	.53
Grade 4:	152	.82	151	.79	.74
Grade 5:	122	.90	116	.87	.50
Grade 6:	128	.87	124	.87	.22
Case 4: No zeros, one occurrence of carrying 1, no occurrences of carrying a number greater than 1					
Grade 1:	103	.03	0	.00	no test
Grade 2:	553	.52	53	.38	2.04
Grade 3:	620	.75	53	.60	2.16
Grade 4:	603	.90	51	.76	2.29
Grade 5:	476	.92	41	.95	-.96
Grade 6:	490	.92	45	.96	-1.16

(Table A13, cont.)

	No "no add" columns		One or two "no add" columns		t value
	N ^a	p ^b	N	p	
Case 5: No zeros, no occurrences of carrying 1, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	155	.37	0	.00	no test
Grade 3:	363	.63	159	.60	.65
Grade 4:	350	.85	152	.80	1.40
Grade 5:	276	.85	123	.80	1.14
Grade 6:	299	.92	129	.85	1.89

Note. There was no passing. There were 2 or 3 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A14

Comparison of Performance on Addition Exercises
with No Carrying and One Occurrence of Carrying 1

	No carrying		One occurrence of carrying 1		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros, no "no add" columns, no occurrences of carrying a number greater than 1					
Grade 1:	257	.40	103	.03	10.60
Grade 2:	255	.81	553	.52	9.03
Grade 3:	255	.91	620	.75	6.17
Grade 4:	251	.94	603	.90	1.83
Grade 5:	197	.96	476	.92	2.04
Grade 6:	204	.97	490	.92	2.75
Case 2: Passing 1 or 2, no zeros, no "no add" columns, no occurrences of carrying a number greater than 1					
Grade 1:	207	.39	0	.00	no test
Grade 2:	206	.76	412	.47	7.60
Grade 3:	213	.88	472	.71	5.73
Grade 4:	204	.92	455	.82	3.58
Grade 5:	161	.93	365	.90	1.21
Grade 6:	175	.97	383	.92	2.61
Case 3: No passing, one occurrence of N + 0, no occurrences of carrying a number greater than 1					
Grade 1:	104	.27	0	.00	no test
Grade 2:	208	.69	99	.61	1.36
Grade 3:	209	.88	104	.76	2.54
Grade 4:	200	.92	100	.91	.30
Grade 5:	162	.94	82	.87	1.88
Grade 6:	176	.95	77	.94	.39
Case 4: No passing, no zeros, 1 or 2 "no add" columns, no occurrences of carrying a number greater than 1					
Grade 1:	156	.20	0	.00	no test
Grade 2:	155	.50	53	.38	1.51
Grade 3:	157	.65	53	.60	.65
Grade 4:	152	.82	0	.00	no test
Grade 5:	118	.87	41	.95	-1.82
Grade 6:	128	.94	45	.96	-.57

(Table A14, cont.)

	No carrying		One occurrence of carrying 1		t value
	N ^a	p ^b	N	p	
Case 5: No zeros, no "no add" columns, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	155	.37	0	.00	no test
Grade 3:	363	.63	259	.64	-.14
Grade 4:	150	.90	51	.76	2.16
Grade 5:	276	.85	197	.82	.65
Grade 6:	299	.92	198	.85	2.36

Note. There were no occurrences of 0 + N. There were 2 or 3 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A15

Comparison of Performance on Addition Exercises
with No Carrying and One Occurrence of Carrying a Number Greater than 1

	No carrying		One occurrence of carrying a number greater than 1		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros, no "no add" columns					
Grade 1:	257	.40	0	.00	no test
Grade 2:	255	.81	155	.37	9.61
Grade 3:	255	.91	363	.63	8.74
Grade 4:	251	.94	350	.85	3.55
Grade 5:	197	.96	276	.85	4.18
Grade 6:	204	.97	299	.92	2.62
Case 2: Passing 1 or 2, no zeros, no "no add" columns					
Grade 1:	207	.39	0	.00	no test
Grade 2:	206	.76	101	.39	6.57
Grade 3:	213	.88	366	.60	8.37
Grade 4:	204	.92	351	.78	4.73
Grade 5:	161	.93	287	.79	4.44
Grade 6:	175	.97	286	.82	5.83
Case 3: No passing, one occurrence of N + 0, no occurrences of "no add" columns					
Grade 1:	104	.27	0	.00	no test
Grade 2:	208	.69	0	.00	no test
Grade 3:	209	.88	102	.63	4.79
Grade 4:	200	.92	101	.80	2.72
Grade 5:	162	.94	76	.87	1.77
Grade 6:	176	.95	85	.86	2.10
Case 4: No passing, no occurrences of N + 0, one occurrence of "no add" columns					
Grade 1:	157	.26	0	.00	no test
Grade 2:	151	.57	0	.00	no test
Grade 3:	159	.68	53	.60	1.03
Grade 4:	152	.82	49	.73	1.32
Grade 5:	122	.90	39	.69	2.59
Grade 6:	128	.87	45	.73	1.99

(Table A15, cont.)

	No carrying		One occurrence of carrying a number greater than 1		t value
	N ^a	p ^b	N	p	
Case 5: No passing, no zeros, 1 or 2 "no add" columns					
Grade 1:	156	.20	0	.00	no test
Grade 2:	155	.50	0	.00	no test
Grade 3:	157	.65	159	.60	.88
Grade 4:	150	.90	152	.80	2.55
Grade 5:	118	.87	123	.80	1.43
Grade 6:	128	.94	129	.85	2.32

Note. There were no occurrences of carrying 1. There were 2 or 3 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A16

Comparison of Performance on Addition Exercises

with $N + 0$ and $0 + N$

	One to two occurrences of N + 0		One occurrence of 0 + N		t value
	N ^a	p ^b	N	p	
Case 1: One column, no passing					
Grade 1:	157	.48	52	.54	-.78
Grade 2:	155	.79	52	.63	2.10
Grade 3:	158	.92	53	.74	2.79
Grade 4:	154	1.00	49	.90	2.33
Grade 5:	122	.98	39	.90	1.68
Grade 6:	134	.95	45	1.00	-2.73
Case 2: Two or three columns, no passing, no "no add" columns, no carrying					
Grade 1:	104	.27	157	.26	.18
Grade 2:	208	.69	151	.57	2.33
Grade 3:	209	.88	159	.68	4.66
Grade 4:	200	.92	152	.82	2.67
Grade 5:	162	.94	122	.90	1.50
Grade 6:	176	.95	128	.87	2.17
Case 3: Two or three columns, passing 1 or 2, no "no add" columns, no carrying					
Grade 1:	104	.29	52	.29	-.02
Grade 2:	304	.56	52	.63	-1.04
Grade 3:	365	.78	53	.72	.90
Grade 4:	351	.85	49	.80	.78
Grade 5:	275	.91	39	.74	2.36
Grade 6:	297	.95	45	.84	1.88
Case 4: Two or three columns, no passing, 1 or 2 "no add" columns, no carrying					
Grade 1:	53	.21	154	.19	.25
Grade 2:	52	.60	153	.49	1.39
Grade 3:	53	.77	154	.65	1.72
Grade 4:	50	.92	151	.79	2.57
Grade 5:	42	.98	116	.87	2.82
Grade 6:	44	.91	124	.87	.86

(Table A16, cont.)

	One to two occurrences of N + 0		One occurrence of 0 + N		t value
	N ^a	p ^b	N	p	
Case 5: Two or three columns, no passing, no "no add" columns, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	0	.00	0	.00	no test
Grade 3:	102	.63	53	.60	.34
Grade 4:	101	.80	49	.73	.94
Grade 5:	76	.87	39	.69	2.14
Grade 6:	85	.86	45	.73	1.72

Note. There were no occurrences of carrying 1. There were 2 to 4
addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A17

Comparison of Performance on Addition Exercises
with Passing and Carrying

	Passing 1		Carrying 1		t value
	N ^a	p ^b	N	p	
Case 1: No zeros, no occurrences of carrying a number greater than 1					
Grade 1:	207	.39	103	.03	9.47
Grade 2:	206	.76	553	.52	6.49
Grade 3:	213	.88	620	.75	4.70
Grade 4:	204	.92	603	.90	.64
Grade 5:	161	.93	476	.92	.70
Grade 6:	175	.97	490	.92	2.61
Case 2: One occurrence of N + 0, no occurrences of carrying a number greater than 1					
Grade 1:	104	.29	0	.00	no test
Grade 2:	252	.55	99	.61	-.89
Grade 3:	312	.78	104	.76	.43
Grade 4:	302	.84	100	.91	-2.10
Grade 5:	236	.90	82	.87	.90
Grade 6:	252	.95	77	.94	.56
Case 3: No zeros, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	101	.39	0	.00	no test
Grade 3:	314	.61	259	.64	-.81
Grade 4:	301	.78	249	.74	1.21
Grade 5:	245	.81	197	.82	-.29
Grade 6:	248	.81	198	.85	-1.10

Note. There were no occurrences of 0 + N and no "no add" columns. There were 2 or 3 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A18

Comparison of Performance on Addition Exercises
with Carrying 1 and Carrying a Number Greater Than 1

	One occurrence of carrying 1		One occurrence of carrying a number greater than 1		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	256	.51	155	.37	2.70
Grade 3:	309	.77	363	.63	3.94
Grade 4:	302	.92	350	.85	3.05
Grade 5:	233	.91	276	.85	2.41
Grade 6:	255	.94	299	.92	.98
Case 2: Passing 1 or 2, no zeros					
Grade 1:	0	.00	0	.00	no test
Grade 2:	207	.49	101	.39	1.66
Grade 3:	264	.71	366	.60	2.90
Grade 4:	253	.79	351	.78	.43
Grade 5:	204	.86	287	.79	1.82
Grade 6:	217	.90	286	.82	2.54
Case 3: No passing, one occurrence of N + 0					
Grade 1:	0	.00	0	.00	no test
Grade 2:	99	.61	0	.00	no test
Grade 3:	104	.76	102	.63	2.08
Grade 4:	100	.91	101	.80	2.24
Grade 5:	82	.87	76	.87	-.05
Grade 6:	77	.94	85	.86	1.57

Note. There were no occurrences of 0 + N and "no add" columns. There were 2 or 3 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.

Table A19

Comparison of Performance on Addition Exercises
with No Short Columns and Short Columns That Are Not "No Add" Columns

	No short columns		One short column ^a		t value
	N ^a	p ^b	N	p	
Case 1: No passing, no zeros, no carrying					
Grade 1:	153	.40	104	.39	.11
Grade 2:	153	.85	102	.76	1.76
Grade 3:	156	.91	99	.90	.25
Grade 4:	150	.93	101	.94	-.21
Grade 5:	120	.94	77	.97	-1.12
Grade 6:	121	.96	83	.99	-1.55
Case 2: Passing 1 or 2, no zeros, no carrying					
Grade 1:	101	.39	106	.39	-.05
Grade 2:	103	.77	103	.76	.17
Grade 3:	106	.89	107	.87	.54
Grade 4:	102	.92	102	.91	.25
Grade 5:	120	.94	41	.90	.83
Grade 6:	85	.98	90	.96	.43
Case 3: No passing, one occurrence of N + 0, no carrying					
Grade 1:	104	.27	0	.00	no test
Grade 2:	156	.70	52	.63	.97
Grade 3:	156	.88	53	.89	-.24
Grade 4:	150	.91	50	.94	-.65
Grade 5:	120	.97	42	.88	1.65
Grade 6:	132	.95	44	.93	.55
Case 4: No passing, no zeros, one occurrence of carrying 1, no occurrences of carrying a number greater than 1					
Grade 1:	103	.03	0	.00	no test
Grade 2:	450	.52	103	.54	-.27
Grade 3:	462	.74	158	.77	-.57
Grade 4:	453	.90	150	.91	-.65
Grade 5:	356	.91	120	.93	-.87
Grade 6:	361	.93	129	.92	.17

(Table A19, cont.)

	No short columns		One short column ^a		t value
	N ^a	p ^b	N	p	
Case 5: No passing, no zeros, no occurrences of carrying 1, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	103	.42	52	.29	1.58
Grade 3:	209	.61	154	.67	-1.14
Grade 4:	199	.84	151	.87	-.83
Grade 5:	157	.82	119	.88	-1.34
Grade 6:	175	.90	124	.94	-1.19

Note. There were no occurrences of 0 + N, no "no add" columns. There were 2 or 3 columns and 2 to 4 addends.

^aNot a "no add" column.

^bN = number of children given the exercises.

^cp = observed proportion of correct responses.

Table A20

Comparison of Performance on Addition Exercises
with Short Columns That Are Not "No Add" Columns and "No Add" Columns

	Short columns that are not "no add" columns		"No add" columns		t value
	N ^a	p ^b	N	p	
Case 1: No zeros, no carrying					
Grade 1:	104	.39	156	.20	3.30
Grade 2:	102	.76	155	.50	4.50
Grade 3:	99	.90	157	.65	5.14
Grade 4:	101	.94	150	.90	1.18
Grade 5:	77	.97	118	.87	2.99
Grade 6:	83	.99	128	.94	2.12
Case 2: One occurrence of N + 0, no carrying					
Grade 1:	0	.00	53	.21	no test
Grade 2:	52	.63	52	.60	.32
Grade 3:	53	.89	53	.77	1.67
Grade 4:	50	.94	50	.92	.39
Grade 5:	42	.88	42	.98	-1.83
Grade 6:	44	.93	44	.91	.35
Case 3: No zeros, one occurrence of carrying 1, no occurrences carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	103	.54	53	.38	1.86
Grade 3:	158	.77	53	.60	2.21
Grade 4:	150	.91	51	.76	2.39
Grade 5:	120	.93	41	.95	-.42
Grade 6:	129	.92	45	.96	-1.06
Case 4: No zeros, no occurrences of carrying 1, one occurrence of carrying a number greater than 1					
Grade 1:	0	.00	0	.00	no test
Grade 2:	52	.29	0	.00	no test
Grade 3:	154	.67	159	.60	1.17
Grade 4:	151	.87	152	.80	1.65
Grade 5:	119	.88	123	.80	1.71
Grade 6:	124	.94	129	.85	2.32

Note. There was no passing and no occurrences of 0 + N. There were 2 or 3 columns and 2 to 4 addends.

^aN = number of children given the exercises.

^bp = observed proportion of correct responses.